Schutz

7.3. $g_{\alpha\beta} = \begin{pmatrix} -(1+2\phi) & & & \\ & 1-2\phi & & \\ & & 1-2\phi & \\ & & & 1-2\phi \end{pmatrix}$, $g^{\alpha\beta} = \begin{pmatrix} -(1-2\phi) & & & \\ & 1+2\phi & & \\ & & 1+2\phi & \\ & & & 1+2\phi \end{pmatrix}$.

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\sigma\lambda} [g_{\sigma\mu,\nu} + g_{\sigma\nu,\mu} - g_{\mu\nu,\sigma}].$$

$$\Rightarrow \Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\lambda} [g_{\lambda\mu,\nu} + g_{\lambda\nu,\mu} - g_{\mu\nu,\lambda}]$$

$\mu\nu$ have independent components of $\begin{bmatrix} 00 & 01 & 02 & 03 \\ & 11 & 12 & 13 \\ & & 22 & 23 \\ & & & 33 \end{bmatrix}$ due to symmetry constraint.

$$\Gamma^{0}_{00} = \frac{1}{2}(-(1-2\phi))(-(1+2\phi_{,0}))$$

$$= \frac{1}{2}[1-2\phi][1+2\phi_{,0}]$$

$$\Gamma^{0}_{01} = \frac{1}{2}[-(1-2\phi)][-(1+2\phi_{,1})]$$

$$= \frac{1}{2}[1-2\phi][1+2\phi_{,1}].$$

by symmetry, we also have $\Gamma^{0}_{02}, \Gamma^{0}_{03}$ $= \frac{1}{2}[1-2\phi][1+2\phi_{,2}]$ and $\frac{1}{2}[1-2\phi][1+2\phi_{,3}]$.

$$\Gamma^{1}_{11} = \frac{1}{2}[-(1-2\phi)][-(1-2\phi_{,0})]$$

$$= \frac{1}{2}[1-2\phi][1-2\phi_{,0}],$$

$$\Gamma^{0}_{12} = \Gamma^{0}_{13} = 0.$$

Similarily, by symmetry, we will also have

$$\Gamma^0_{22} = \frac{1}{2}[1-2\phi][1-2\phi_{,0}] = \Gamma^0_{33}$$

and it's easy to see for similar reasons, $\Gamma^0_{23} = 0$.

$$\Gamma^1_{00} = \frac{1}{2}\cancel{\phi}[1+2\phi][-(-(1+2\phi_{,1}))]$$

$$= \frac{1}{2}[1-2\phi][1+2\phi_{,1}].$$

$$\Gamma^1_{01} = \frac{1}{2}[1+2\phi][1-2\phi_{,0}], \quad \Gamma^1_{02} = \Gamma^1_{03} = 0.$$

$$\Gamma^1_{11} = \frac{1}{2}[1+2\phi][1-2\phi_{,1}]$$

$$\Gamma^1_{12} = \frac{1}{2}[1+2\phi][1-2\phi_{,2}]$$

$$\Gamma^1_{13} = \frac{1}{2}[1+2\phi][1-2\phi_{,3}],$$

$$\Gamma^1_{22} = \frac{1}{2}[1+2\phi][1-2\phi_{,1}][-1],$$

$$\Gamma^1_{33} = \frac{1}{2}[1+2\phi][1-2\phi_{,1}][-1].$$

Notice Γ^1_{11}, Γ^1_{12}, Γ^1_{13}, Γ^1_{22}, Γ^1_{33} are independent of time, by spatial symmetry, we apply a permutation $1 \to 2$, $2 \to 3$, $3 \to 1$ to obtain

$$\Gamma^2_{22} = \frac{1}{2}[1+2\phi][1-2\phi,2], \quad \Gamma^2_{23} = \frac{1}{2}[1+2\phi][1-2\phi,3],$$

$$\Gamma^2_{31} = \frac{1}{2}[1+2\phi][1-2\phi,1], \quad \Gamma^2_{33} = \frac{1}{2}[1+2\phi][1-2\phi,2][-1],$$

$$\Gamma^2_{11} = \frac{1}{2}[1+2\phi][1-2\phi,2][-1].$$

Applying $1\to 2$, $2\to 3$, $3\to 1$ again yields.

$$\Gamma^3_{33} = \frac{1}{2}[1+2\phi][1-2\phi,3], \quad \Gamma^3_{31} = \frac{1}{2}[1+2\phi][1-2\phi,1],$$

$$\Gamma^3_{12} = \frac{1}{2}[1-2\phi][1-2\phi,2], \quad \Gamma^3_{11} = \frac{1}{2}[1+2\phi][1-2\phi,3](-1).$$

$$\Gamma^3_{22} = \frac{1}{2}[1+2\phi][1-2\phi,3](-1).$$

It only remains to determine Γ^2_{00}, Γ^3_{00}, Γ^2_{02}, Γ^3_{02} explicitly:

$$\Gamma^2_{00} = \frac{1}{2}[1-2\phi][1+2\phi,2], \quad \Gamma^3_{00} = \frac{1}{2}[1-2\phi][1+2\phi,3],$$

$$\Gamma^2_{02} = \frac{1}{2}[1+2\phi][1-2\phi,0] = \Gamma^3_{03}.$$